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A Note on Valuation in Private Equity Settings

The valuation of private companies, especially those in the earlier stages of their life-cycle, is a difficult and often subjective process. Early-stage companies typically forecast a period of negative cash flows with highly uncertain—but tantalizing—future rewards. This cash flow profile is very sensitive to the valuation assumptions made.

This note will discuss a variety of valuation techniques that can be used in private equity settings. The intention is to provide a practical tool-kit to be used when tackling cases in the second and third modules of "Venture Capital and Private Equity." Much of the background theory will be glossed over: the focus will be on the essential underlying mechanics of each method and a discussion of strengths and weaknesses. The references at the end of this note provide more detailed information on the various valuation techniques discussed.

This note addresses the Comparables, Net Present Value, Adjusted Present Value, Venture Capital, and Options valuation methods. We also discuss the use of Monte Carlo simulation employing the software package Crystal Ball® to enhance these valuations. The following five sections are each dedicated to one valuation method. Each section has a corresponding appendix with a detailed example of the method.

1. Comparables

The use of comparables often provides a quick and easy way to obtain a "ballpark" valuation for a firm. When searching for comparables, we seek other firms that display similar "value characteristics" to the company we are interested in. These value characteristics include risk, growth rate, capital structure, and the size and timing of cash flows. Often, these value characteristics are driven by other underlying attributes of the company which can be incorporated in a multiple. For example, the anticipated cash flows for a new Health Maintenance Organization (HMO) might be accurately predicted by the number of members it has enrolled (see the example in **Appendix 1**).

There are, however, many potential problems with the use of comparables for private companies. (The strengths and weaknesses of each method are summarized in **Exhibit 1**.) First, it is often difficult to ascertain what valuations have been assigned to other privately held firms. It may consequently be impossible to compare our firm to the companies that are the most similar. Second, because accounting and other performance information on private firms are often unavailable, key ratios may not be calculable, or other important impacts on valuation may be missed. Finally, the

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valuations assigned to comparable firms may be misguided. Periodically, whole classes of firms have been valued at prices that seem unjustifiable on a cash flow basis.

Sound judgment should consequently drive the use of comparables One must search for potential measures of value that can be sensibly applied from one company to the next. In public markets, common ratios are (i) the share price divided by the earnings per share (the P-E ratio), *(ii)* the market value of the firm's equity divided by total revenue, and *(iii)* the market value of the firm's equity divided by the shareholder's equity on the balance sheet (market-to-book ratio). These ratios, however, may be misleading. Consider the price-earnings ratio. The earnings (profit after tax) reflects the capital structure of the company, since earnings are after interest expenses and taxes. Common sense would therefore tell us that when comparing two companies with similar characteristics except for substantially different capital structures, it would be more appropriate to use a multiple based on earnings before interest and taxes (EBIT). By using this latter comparable, we compensate for the differing capital structures of the two entities. This is because EBIT ignores the different levels of interest expense incurred by the two companies. (Of course, the use of EBIT ignores the interest tax shields associated with these capital structures, which we may wish to factor into the comparisons.)

Accounting-based comparables, such as those mentioned above, are clearly less suitable in a private equity setting where companies are often unprofitable and experiencing rapid growth. One must therefore look for other sensible measures of value. For example, in an Internet business a good indicator of value may be the number of subscribers enrolled by a company. A valid proxy for the value of a biotechnology firm may be the number of patents awarded. In a gold exploration company, a typical measure of value is the number of ounces of gold indicated by initial drilling results. These are just a few examples of non-financial, industry-specific measures that can be used to estimate the value of a firm.

Interestingly, a recent study suggests that industry-specific multiples have strong explanatory power for the offering prices of IPOs. In contrast, accounting-based multiples, such as the price-earnings ratio and the ratio of the market to book value of equity, were found to have little predictive ability. The reason is that among young, publicly traded firms in the same industry, accounting-based multiples vary substantially.[1]

Another issue related to the use of public market comparables to value private companies is the marketability of the equity. Because shares in private firms are typically less marketable than those of publicly traded firms, it may be appropriate to apply a discount for illiquidity. The size of the proper discount will depend on the particular circumstances. Surveys suggest, however, that discounts for lack of marketability used in practice fall within a very narrow band, often between 25% and 30%.[2]

2. The Net Present Value Method

The Net Present Value (NPV) method is one of the most common methods of cash flow valuation. (Others include the Equity Cash Flow and Capital Cash Flow methods. The Adjusted Present Value method discussed in the next section is a variation on the Capital Cash Flow method.) This section briefly visits the basics of the NPV method.

[1]M. Kim and J. Ritter, "Valuing IPOs," unpublished working paper, Sloan School of Management, Massachusetts School of Technology, 1996.

[2] S. Pratt, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies* (Homewood, Ill.: Dow Jones-Irwin, 1996).

The NPV method incorporates the benefit of tax shields from tax-deductible interest payments in the discount rate *(i.e.,* the Weighted Average Cost of Capital, or WACC). To avoid double-counting these tax shields, interest payments must not be deducted from cash flows. Equation (1) shows how to calculate cash flows (subscripts denote time periods):

$$CF_t = EBIT_t * (1 - \tau) + DEPR_t - CAPEX_t - \Delta NWC_t + other_t \quad \textbf{(1)}$$

where:

CF = cash flow
EBIT = earnings before interest and tax
τ = corporate tax rate
DEPR = depreciation
CAPEX = capital expenditures
ΔNWC = increase in net working capital
other = increases in taxes payable, wages payable, etc.

Next, the terminal value should be calculated. This estimate is very important as the majority of the value of a company, especially one in an early-stage setting, may be in the terminal value. A common method for estimating the terminal value of an enterprise is the perpetuity method.

Equation (2) gives the formula for calculating a terminal value (TV) at time T using the perpetuity method, assuming a growth rate in perpetuity of g and a discount rate equal to r. The cash flows and discount rates used in the NPV method are typically nominal values *(i.e.,* they are not adjusted for inflation). If forecasts indicate that the cash flow will be constant in inflation-adjusted dollars, a terminal growth rate equal to the rate of inflation should be used:

$$TV_T = [CF_T * (1 + g)] / (r - g) \quad \textbf{(2)}$$

Other common methods of terminal value calculation used in practice include price-earnings ratios and market-to-book value multiples, but these short-cuts are not encouraged!

The net present value of the firm is then calculated as shown in Equation (3):

$$NPV = [CF_1 / (1 + r)] + [CF_2 / (1 + r)^2] + [CF_3 / (1 + r)^3] + \ldots + [(CF_T + TV_T) / (1 + r)^T] \quad \textbf{(3)}$$

The discount rate is calculated using Equation (4):

$$r = (D/V) * r_d * (1 - t) + (E/V) * r_e \quad \textbf{(4)}$$

where:

r_d = discount rate for debt
r_e = discount rate for equity
τ = corporate tax rate
D = market value of debt
E = market value of equity
V = D + E

If the firm is not at its target capital structure, however, the target values should be used for D/V and E/V.

The cost of equity re is calculated using the familiar Capital Asset Pricing Model shown in Equation (5):

$$r_e = r_f + \beta * (r_m - r_f) \quad \textbf{(5)}$$

where:

r_e	= discount rate for equity
r_f	= risk-free rate
β	= beta, or degree of correlation with the market
r_m	= market rate of return on common stock
$(r_m - r_f)$	= market risk premium

When determining the appropriate risk free rate (r_f), one should attempt to match the maturity of the investment project with that of the risk free rate. Typically, we use the ten-year rate. Estimates of the market risk premium can vary widely: for the sake of the course, 7.5% can be assumed.

For private companies, or spin-offs from public companies, betas can be estimated by looking at comparable public firms. The beta for public companies can be found in a beta book or on the Bloomberg machine. If the firm is not at its target capital structure, it is necessary to "unlever" and "relever" the beta. This is accomplished using Equation (6):

$$\beta_u = \beta_t * (E / V) = \beta_t * [E / (E + D)] \quad \textbf{(6)}$$

where:

β_u	= unlevered beta
β_t	= levered beta
E	= market value of equity
D	= market value of debt

An issue arises where there are no comparable companies. This often occurs in entrepreneurial settings. Common sense is the best guide in this situation. Think about the cyclical nature of the particular firm and whether the risk is systematic or can be diversified away. If accounting data is available, another way is to calculate "earnings betas," which have some correlation with equity betas. An earnings beta is calculated by comparing a private company's net income to a stock market index such as the S&P 500. Using least squares regression techniques, the slope of the line of best fit (the beta) can be calculated.

Strengths and Weaknesses of the Net Present Value Method

Estimating firm values by discounting relevant cash flows is widely regarded as technically sound. The values should be less subject than comparables to distortions that can occur in public and, more commonly, private markets.

Given the many assumptions and estimates that have been made during the valuation process, however, it is unrealistic to arrive at a single, or "point," value for the firm. Different cash flows should be estimated under "best," "most likely," and "worst" case assumptions. These should then be discounted using a range of values for WACC and the terminal growth rate (g) to give a likely range of values. If you can assign probabilities to each scenario, a weighted average will determine the expected value of the firm.

Even with these steps, the NPV method still has some drawbacks. First, we need betas to calculate the discount rate. A valid comparable company should have similar financial performance, growth prospects, and operating characteristics to the company being valued. A public company with these characteristics may not exist. On a similar note, the target capital structure is often estimated using comparables. Using comparable companies to estimate a target capital structure has much the same drawbacks as finding comparable betas. Third, the typical start-up company cashflow profile of large initial expenditures followed by distant inflows leads to much (or even all) of the value being in the terminal value. Terminal values are very sensitive to assumptions about both discount and terminal growth rates. Finally, recent finance research has raised questions as to whether beta is the proper measure of firm risk. Numerous studies suggest that firm size or the ratio of book-to-market equity values may be more appropriate.[3] Few have tried to implement these suggestions, however, in a practical valuation context.

Another drawback of the NPV method is in the valuation of companies with changing capital structures or effective tax rates. Changing capital structures are often associated with highly leveraged transactions, such as leveraged buyouts. Changing effective tax rates can be due to the consumption of tax credits, such as net operating losses, or the expiration of tax subsidies sometimes granted to fledgling firms. Under the NPV method, the capital structure and effective tax rate are both incorporated in the discount rate (WACC) and assumed to be constant. For this reason the Adjusted Present Value method (Section 3) is recommended in these cases.

Monte Carlo Simulation

When calculating values using spreadsheets, we arrive at a single, or "point," estimate of value. Even when undertaking sensitivity analysis, we simply alter variables one at a time, and determine the change in the valuations. Monte Carlo simulation is an improvement over simple sensitivity analysis because it considers all possible combinations of input variables. The user defines probability distributions for each input variable, and the program generates a probability distribution describing the possible outcomes.

One such package, which shall be described here and used in class, is Crystal Ball®.[4] The first step is to set up the base case spreadsheet. We then define the assumption and forecast variables: we will determine the effect of changes in the assumption cells on the value contained in the forecast cell. Assumption cells contain variables such as the discount rate, terminal growth rate, and cash flows. Assumption cells must contain numerical values, not formulas or text. Probability distributions are used to define the way in which the values in the assumption cells vary. Crystal Ball® has a suite of probability distributions to choose in describing the behavior of each variable. The user needs to select an appropriate distribution, and estimate the key parameters (e.g., mean and standard deviation).

Assumptions can be defined by highlighting one variable at a time and using the command *Cell Define Assumption.* Similarly, the forecast is defined by highlighting the cell with the valuation calculation and using the command *Cell Define Forecast.* A simulation is then generated using the command *Run Run.* To create a report, use the command *Run Create Report.* A summary of the report for the NPV valuation performed in **Appendix 2** is shown in **Exhibit 2**. It shows the probability distribution for the value of the subsidiary, Hi-Tech. The report also indicates that the assumptions were defined as normal distributions with means equal to the values initially contained in the cells, and standard deviations set at +10% of the mean.

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[3] For an overview, see Eugene F. Fama and Kenneth R. French, "The Cross-Section of Expected Stock Returns," *Journal of Finance*, 47 (1992): 427-465.

[4] Crystal Ball® is a personal computer simulation package produced by Decisioneering, Inc., which is located at 1515 Arapahoe Street, Suite 1311, Denver, CO 80202. Its phone is 800-289-2550 or 303-534-1515; its fax, 303-534-4818; and address on the World Wide Web, http://www.decisioneering.com.

The availability and simplicity of simulation packages make them a useful tool. Simulation allows a more thorough analysis of the possible outcomes than does regular sensitivity analysis. An additional benefit is that simulation packages allow the user to consider the interrelationships between the different input variables: as the manual describes, it is easy to define correlations between the various explanatory variables. One must remember, however, that in reality the shapes of distributions, and interrelationships between variables, can be very hard to discover. As sophisticated as the output reports look, the old adage about a model being only as good as the assumptions behind it still applies.

3. The Adjusted Present Value Method

The Adjusted Present Value (APV) method is a variation of the NPV method. APV is preferred over the NPV method where a firm's capital structure is changing or it has net operating losses (NOLs) that can be used to offset taxable income. (An example demonstrating the APV method can be found in **Appendix 3**.)

The NPV method assumes that the capital structure of the firm remains constant at a prespecified target level. This is inappropriate in situations such as leveraged buyouts, where initially the capital structure is highly leveraged, but the level of debt is reduced as repayments are made. In this case, the "target" capital structure changes over time. A way of illustrating this issue is by considering a LBO firm with an ultimate target capital structure of zero: *i.e.,* after a certain period it aims to have paid off all its debt. Under the NPV method, the discount rate (WACC) would be calculated using an all-equity capital structure. This ignores the fact the firm has been levered up. APV overcomes this drawback by considering the cash flows generated by the assets of a company, ignoring its capital structure. The savings from tax-deductible interest payments are then valued separately.

The NPV method also assumes that the firm's effective tax rate, incorporated in the WACC, remains constant. This is inappropriate where a firm's effective tax rate changes over time. For example, it is typical for a start-up company to have incurred NOLs before it attains profitability. Under certain circumstances, these NOLs can be carried forward for tax purposes and netted against taxable income. APV accounts for the effect of the firm's changing tax status by valuing the NOLs separately.

Under APV, the valuation task is divided into three steps. First, the cash flows are valued, ignoring the capital structure. The cash flows of the firm are discounted in the same manner as under the NPV method, except that a different discount rate is used. We essentially assume that the company is financed totally by equity. This implies that the discount rate should be calculated using an unlevered beta, rather than the levered beta used to compute the WACC used in the NPV analysis. The discount rate is calculated using the Capital Asset Pricing Model shown in Equations (5) and (6).

The tax benefits associated with the capital structure are then estimated. The net present value of the tax savings from tax-deductible interest payments have value to a company and must be quantified. The interest payments will change over time as debt levels are increased or reduced. By convention, the discount rate often used to calculate the net present value of the tax benefits is the pre-tax rate of return on debt. This will be lower than the cost of equity. Conceptually this is sensible. The claims of debt holders rank higher than those of ordinary shareholders and therefore are a safer stream of cash flows.

Finally, NOLs available to the company also have value which must be quantified. NOLs can be offset against pre-tax income and often provide a useful source of cash to a company in its initial profitable years of operation. For instance, if a company has $10 million of NOLs and the prevailing tax rate is 40%, the company will have tax savings of $4 million. (Note, however, that this ignores the time value of money. The net present value of the NOLs will only be $4 million if the firm has taxable income of $10 million in its first year. If the NOLs are utilized over more than one year, then discounting will reduce their value to some amount less than $4 million.)

The discount rate used to value NOLs is often the pre-tax rate on debt. If you believe that the realization of tax benefits from the NOLs is certain, (*i.e.*, the firm will definitely generate sufficient profits to consume them), then use the risk-free rate. If, however, there is some risk that the firm will not generate enough profits to use up the NOLs, then discounting them by the pre-tax rate of corporate debt makes sense.

4. The Venture Capital Method

The Venture Capital method is a valuation tool commonly applied in the private equity industry. As discussed, private equity investments are often characterized by negative cash flows and earnings, and highly uncertain but potentially substantial future rewards. The Venture Capital method accounts for this cash flow profile by valuing the company, typically using a multiple, at a time in the future when it is projected to have achieved positive cash flows and/or earnings. This "terminal value" is then discounted back to the present using a high discount rate, typically between 40% and 75%. (The rationales for these very high target rates are discussed below.)

The venture capitalist uses this discounted terminal value and the size of the proposed investment to calculate her desired ownership interest in the company. For example, if the company's discounted terminal value is $10 million, and the venture capitalist intends to make a $5 million investment, she will want 50% of the company in exchange for her investment. This assumes, however, that there will be no dilution of the venture capitalist's interest through future rounds of financing. This is an unrealistic assumption, given that most successful venture-backed companies sell shares to the public through an IPO.

The underlying mechanics of the Venture Capital method are demonstrated by the following four steps. (An example demonstrating the Venture Capital method can be found in **Appendix 4**.) The method starts by estimating the company's value in some future year of interest, typically shortly after the venture capitalist foresees taking the firm public. The "terminal value" is usually calculated using a multiple: for example, a price-earnings ratio may be multiplied by the projected net income in the exit year. (See the discussion of comparables in Section 1.) The Terminal Value can of course be calculated using other techniques, including discounted cash flow methods.

The Discounted Terminal Value of the company is determined by, not surprisingly, discounting the Terminal Value calculated in the first step. Instead of using a traditional cost of capital as the discount rate, however, venture capitalists typically use a Target Rate of Return. The Target Rate of Return is the yield the venture capitalist feels is required to justify the risk and effort of the particular investment. The formula for calculating the Discounted Terminal Value is shown in Equation (7):

$$\textit{Discounted Terminal Value} = \textit{Terminal Value} / (1 + \textit{Target})^{years} \quad \textbf{(7)}$$

Third, the venture capitalist calculates the Required Final Percent Ownership. The amount of the proposed investment is divided by the Discounted Terminal Value to determine the ownership necessary for the venture capitalist to earn her desired return (assuming that there is no subsequent dilution of her investment):

Required Final Percent Ownership = Investment / Discounted Terminal Value **(8)**

Finally, she estimates future dilution and calculates the required current percent ownership. Equation (8) would be the correct answer if there were to be no subsequent "rounds" of financing to dilute the venture capitalist's interest in the company. As we have seen in the course, venture-backed companies commonly receive multiple rounds of financing, followed by an IPO. Hence, this assumption is usually unrealistic. To compensate for the effect of dilution from future rounds of financing, she needs to calculate the Retention Ratio. The Retention Ratio quantifies the expected dilutive effect of future rounds of financing on the venture capitalist's ownership. Consider a firm that intends to undertake one more financing round, in which shares representing an additional 25% of the firm's equity will be sold, and then to sell shares representing an additional 30% of the firm at the time of the IPO. If the venture capitalist owns 10% today, after these financings her stake will be 10%/(1+. 25)/(1+.3) = 6.15%. Her retention ratio is 6.15%/10% = 61.5%.

The Required Current Percent Ownership necessary for the venture capitalist to realize her Target Rate of Return is then calculated using Equation (9):

Required Current Percent Ownership = Required Final Percent Ownership / Retention Ratio **(9)**

Strengths and Weaknesses of the Venture Capital Method

A major criticism of the Venture Capital method is the use of very large discount rates, typically between 40% and 75%. Venture capitalists justify the use of these high target returns on a number of grounds. First, they argue that large discount rates are used to compensate for the illiquidity of private firms. As discussed in Section 1, equity of private companies is usually less marketable than public stock, and investors demand a higher return in exchange for this lack of marketability. Second, venture capitalists view their services as valuable and consider the large discount rate as providing compensation for their efforts. For example, they provide strategic advice, credibility, and access to specialized intermediaries such as lawyers and investment bankers. Finally, venture capitalists believe that projections presented by entrepreneurs tend to be overly optimistic. They submit that the large discount rate compensates for these inflated projections.

Financial economists suggest that although the issues raised by venture capitalists may be valid, they should not be addressed through a high discount rate. They propose that each of the "justifications" should be valued separately using more objective techniques. First, they argue that the discount for lack of marketability makes sense, but that the estimated premium is far too large: there are numerous investors with long-run time horizons, including endowments, foundations, and individuals. Second, financial economists contend that the services provided by the venture capitalist should be valued by determining what that would have to be paid to acquire equivalent professional services on a contract basis. Once the fair market value of the services provided was determined, shares equal to this value could be given to the venture capitalist. Finally, financial economists submit that discount rates should not be inflated to compensate for the entrepreneurs' overly optimistic projections. They argue that judgment should be applied to determine the likely values of various scenarios and the probability that they will occur. This will result in unbiased estimates of the cash flow of the firm.

The use of high discount rates suggests an element of arbitrariness in the venture capitalist's approach to valuing a company. A better process is to scrutinize the projections and perform reality checks. This involves asking a number of questions. What has been the performance of comparable companies? What share of the market does the company need to meet its projections? How long will it take? What are the key risks? Are contingency plans in place? What are the key success factors?

This type of analysis is far more meaningful than just taking the entrepreneur's pro formas and discounting them at a very large rate.

5. Options Analysis

In some cases, it is appropriate and desirable to use option pricing techniques to value investment opportunities. Discounted cash flow methods such as NPV and APV can be deficient in situations where a manager or investor has "flexibility." Flexibility can take many forms, including the ability to increase or decrease the rate of production, defer development, or abandon a project. These changes all affect the value of the firm in ways that are not accurately measured using discounted cash flow techniques. One form of flexibility that is of particular interest to the venture capitalist is the ability to make "follow-on" investments.

Private equity-backed companies are often characterized by multiple rounds of financing. Venture capitalists use this multi-stage investment approach to motivate the entrepreneur to "earn" future rounds of financing and also to limit the fund's exposure to a particular portfolio company. Often, the first right of refusal for a later stage of financing is written into the investment contract.

The right to make a follow-on investment has many of the same characteristics as a call option on a company's stock. Both comprise the right, but not the obligation, to acquire an asset by paying a sum of money on or before a certain date. As we shall see, this flexibility is not readily accounted for by discounted cash flow techniques. By way of contrast, option pricing theory accounts for the manager's ability to "wait and then decide whether to invest" in the project at a later date.

To illustrate the drawback of using NPV flow methods when pricing options, consider the following simplified example. A project requiring an investment of $150 today is equally likely to generate revenues next year, that—discounted to today's dollars—total $200, $160, or $120. Consequently, the project will have a net present value of $50, $10, or –$30. The expected return is $10 [= (1/3) * (50 + 10 – 30)].

Now consider an investor who has the ability to delay his investment until period 1.[5] By delaying investing until he obtains further information, he can avoid investing when revenues will only be $120. Essentially, by waiting and gathering more information, the investor modifies the expected return profile from [$50, $10, –$30] to [$50, $10, $0]. The option to delay investing is worth $10, the difference between the new expected NPV of $20 [= (1/3) * (50 + 10 + 0)] and the earlier $10 expected value.

This section introduces a developing area in finance. For the purposes of brevity a basic knowledge of option pricing theory (at the level, for instance, of Brealey and Myers) is assumed. Readers are referred to the references at the end of this note for further literature on option pricing techniques.

Valuing Firms as Options

The Black-Scholes model values European options using five variables as inputs. For an option on a stock, these comprise the exercise price (X), the stock price (S), the time to expiration (t), the standard deviation (or volatility) of returns on the stock (σ), and the risk free rate (r_f). Using these

[5] We assume the net present value of the investment in today's dollars is still $150, whether the investment is made in Period 0 or Period 1.

variables, we can value the right to buy a share a stock at some future point. We can evaluate a firm's decision to invest in a project using a similar framework. The equivalents are shown in **Table A**.

Table A Financial and Firm Option Variables

Variable	Financial Option	Firm Option
X	Exercise price	Present value of the expenditures required to undertake the project
S	Stock price	Present value of the expected cash flows generated by the project
t	Time to expiration	The length of time that the investment decision can be deferred
σ	Standard deviation of returns on the stock	Riskiness of the underlying assets
r_f	Time value of money	Risk free rate of return

Once the input variables have been estimated, the value of the option can be calculated using a Black-Scholes computer model or a call option valuation table.

Reducing Complex Problems to Options Analyses

Real-world decisions can be difficult to reduce to mathematically solvable problems. There is often great value, however, in attempting to simplify these types of problems. For example, the right to abandon the development of a gold mine is similar to a put option. A finance lease gives the lease holder both the right to cancel the lease by paying a fee (a put option), and the right to purchase the asset for a fixed price at the end of the lease (a call option). This note will consider only the solution of call options using the Black-Scholes formula for European options (which can only be exercised at the end of the period).

Table A describes the five inputs necessary to value an investment option by a firm. The approximation of four of the variables (X, S, t, r_f) is fairly intuitive and is illustrated in the example in **Appendix 5**. The process of estimating the fifth variable, the standard deviation (σ), merits further discussion. One way to estimate the standard deviation is to look at the stock price volatility for businesses with assets comparable to the project or company under consideration. These are, for instance, available on the Bloomberg machine. An important point is that volatilities estimated using this method will require adjustment to take into account the leverage of the comparable company. Remember that leverage amplifies risk, and hence comparable companies with higher leverage than the project under consideration will have higher risk. As a guide, volatilities of 20% to 30% are not unusually high for single companies, and many small technology companies have volatilities of between 40% and 50%.

Strengths and Weaknesses of Using Option Pricing to Value Investment Opportunities

Option pricing theory is useful in situations where there is the "flexibility" to wait, learn more about the prospects of the proposed investment, and then decide whether to invest. As discussed, opportunities that incorporate flexibility will consistently be undervalued using discounted cash flow techniques.

There are at least three concerns associated with the use of option pricing methodology. First, it is not well known to many business people, particularly in the private equity community. As with most "new technologies," it may be difficult to convince associates and counter-parties that its use is valid. A second drawback of the option pricing methodology is the difficulty of reducing real-world opportunities to simple problems that can be valued. While the models can accommodate cases where the firm pays dividends or where the option can be exercised early, the calculations may be more complex. Option pricing used inappropriately can inflate values achieved using other methods, thereby falsely justifying projects that would otherwise be rejected. Finally, some situations may not be appropriate for the Black-Scholes formula. For instance, the exact pricing of a series of call options that are nested (*i.e.*, where one cannot be exercised before the other one is) is a difficult problem. In these cases, it may be best to use simulation techniques.

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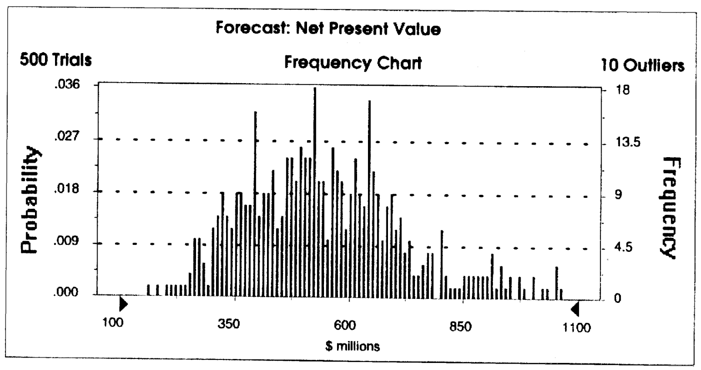
Exhibit 1 Strengths and Weaknesses of Various Valuation Methods in Private Equity Settings

Method	Strengths	Weaknesses
1. Comparables	• Quick to use • Simple to understand • Commonly used in industry • Market based	• Private company comparables may be difficult to find and evaluate • If use public company comparables, need to adjust resulting valuation to take into account private company's illiquidity
2. Net Present Value	• Theoretically sound	• Cash flows may be difficult to estimate • Private company comparables (β and capital structure) can be difficult to find and evaluate • WACC assumes a constant capital structure • WACC assumes a constant effective tax rate • Typical cash flow profile of outflows followed by distant, uncertain inflows is very sensitive to discount and terminal growth rate assumptions
3. Adjusted Present Value	• Theoretically sound • Suitable (and simple to use) in situations where the capital structure is changing (*e.g.*, highly leveraged transactions such as leveraged buyouts) • Suitable in situations where the effective tax rate is changing (*e.g.*, when there are NOLs)	• More complicated to calculate than the NPV method • Same disadvantages as NPV Method except overcomes the shortfalls of the WACC assumption (*i.e.*, constant capital structure and tax rate)
4. Venture Capital	• Simple to understand • Quick to use • Commonly used	• Relies on terminal values derived from other methods • Oversimplified (large discount rate "fudge factor")
5. Asset Options	• Theoretically sound • Overcomes drawbacks of NPV and APV techniques in situations where managers have flexibility	• Methodology is not commonly used in industry and may not be under-stood • Real world situations may be difficult to reduce to solvable option problems • Limitations of Black-Scholes model

Exhibit 2 Simulation Report Produced by Crystal Ball Using Data from **Appendix 2**

Statistics:	Value
Trials	500
Mean	562
Median	535
Mode	---
Standard Deviation	194
Variance	37485
Skewness	0.89
Kurtosis	4.05
Coeff. of Variability	0.34
Range Minimum	162
Range Maximum	1296
Range Width	1134
Mean Std. Error	8.66

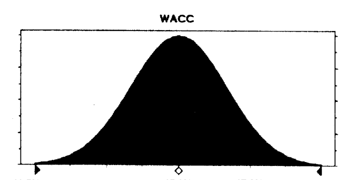


Assumptions

Assumption: WACC

Normal distribution with parameters:

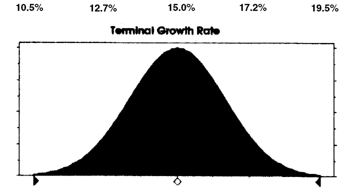
Mean	15.0%
Standard Dev.	1.5%



Assumption: Terminal Growth Rate

Normal distribution with parameters:

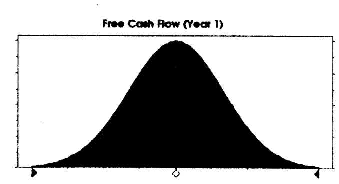
Mean	3.0%
Standard Dev.	0.3%



Assumption: Free Cash Flow (Year 1)

Normal distribution with parameters:

Mean	-140
Standard Dev.	14



Appendix 1
Sample Valuation Using Comparables

The 50-year-old chairman and major shareholder of Private Health, a private regional health maintenance organization (HMO), is considering selling his stake in the company and retiring. He has asked Private Health's chief financial officer (CFO) to calculate the value of the firm by the following morning. The two main options that he is entertaining are the sale of his interest to an Employee Share Ownership Plan (ESOP) and to one of the firm's publicly traded competitors. The CFO regularly receives research reports from investment bankers eager to take the company public. From these reports she is able to compare the following information for Private Health and two public HMOs operating in the same region. Data is for the 1995 financial year (amounts in millions of dollars unless indicated):

	Private Health	Happy Healthcare	Community Health
Balance Sheet			
Assets	160	300	380
Long-Term Debt	5	100	0
Net Worth	80	120	175
Income Statement			
Revenues	350	420	850
EBITDA	45	55	130
Net Income	30	20.0	75.0
Market Data			
Earnings per Share ($/share)	3.00	0.67	2.14
Price-Earnings Ratio (times)	n/a	21.0	14.5
Shares Outstanding (m)	10	30	35
Number of Members	500,000	600,000	1,100,000

From the above information, the CFO was able to calculate the following multiples and implied valuations for Private Health:

	Happy Healthcare	Community Health	Average	Private Health Implied Value ($M)
Price-Earnings Ratio	21.0	14.5	17.7	533
Market Value/EBITDA	7.64	8.37	8.00	360
Market Value/Sales	1.00	1.28	1.14	399
Market Value/Book Value of Equity	3.52	6.21	4.86	389
Market Value/Member	700	989	844	422

The CFO felt that on an overall basis the multiples gave a good indication of the value of Private Health, but that it was overvalued on a P/E multiple basis. She believed this was because Happy Healthcare (long-term debt to total assets of 33%) was substantially more leveraged than Private Health (3%). Valuing Private Health using Community Health's P/E ratio of 14.5 gave an implied valuation of $435 million. Based on her analysis, she was confident that the value of Private Health was in the range of $360-$435 million if sold to a public company. If the shares were sold to an ESOP she believed that, because of the company's private status, it would be appropriate to assume a discount of 15%-20%, or a valuation of $290-$360 million.

Appendix 2
Sample Valuation Using the Net Present Value Method

Lo-Tech's shareholders have voted to cease its diversification strategy and refocus on its core businesses. As a part of this process the company is seeking to divest Hi-Tech, its start-up high-technology subsidiary. George, a venture capitalist, has been approached by the management of Hi-Tech, who want to purchase the company. He decides to value Hi-Tech using the NPV method. George and Hi-Tech management have agreed on the following projections (all data are in millions of dollars):

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
Revenues	100	140	210	250	290	380	500	650	900
Costs	230	240	260	275	290	310	350	400	470
EBIT	–130	–100	–50	–25	0	70	150	250	430

The company has $100 million of NOLs that can be carried forward and offset against future income. In addition, Hi-Tech is projected to generate further losses in its early years of operation that it will also be able to carry forward. The tax rate is 40%. The average unlevered beta of five comparable high-technology companies is 1.2. Hi-Tech has no long-term debt. Treasury yields for ten-year bonds are 6.0%. Capital expenditure requirements are assumed to be equal to depreciation. The market risk premium is assumed to be 7.5%. Net working capital requirements are forecast as 10% of sales. EBIT is projected to grow at 3% per year in perpetuity after Year 9.

George first calculated the Weighted Average Cost of Capital (WACC):

$$WACC = (D / V) * r_d * (1 - \tau) + (E / V) * r_e = 0 + 100\% * [6.0 + 1.2 * (7.5)] = 15\%$$

He then valued the cash flows, which showed the company had a net present value of $525 million. As suspected, all the value of the company was accounted for in the terminal value (the present value of the cash flows was $(44) million and the present value of the terminal value $569 million, giving a net present value of $525 million).

The terminal value was calculated as follows:

$$TV_T = [CF_T * (1 + g)] / (r - g) = [233 * (1 + 3\%)] / (15\% - 3\%) = \$2,000$$

George also performed a scenario analysis to determine the sensitivity of the value of Hi-Tech to changes in the discount rate and the terminal growth rate. He developed a scenario table[6] shown in the attached spreadsheet.

George's scenario analysis gave a series of values ranging from $323 million to $876 million. Clearly this large range did not provide precise guidance as to Hi-Tech's actual value. He noted that the cash flow profile of negative early cash flows followed by distant positive cash flows made the valuation very sensitive to both the discount rate and the terminal growth rate. George considered the NPV method a first step in the valuation process and planned to use other methods to narrow the range of possible values for Hi-Tech.

[6] Sensitivity analyses can be easily undertaken using the Microsoft Excel command *Data Table.*

Appendix 2 (continued)

WACC Calculation	
Tax Rate	40%
Rm – Rf	7.5%
E/V	100%
Bu	1.2
10 Year Treasury Bond	6.0%
WACC	15.0%

Cash Flows

Terminal Growth Rate 3.0%

Year	0	1	2	3	4	5	6	7	8	9
Revenues		100	140	210	250	290	380	500	650	900
Less: Costs		230	240	260	275	290	310	350	400	470
EBIT		–130	–100	–50	–25	0	70	150	250	430
Less: Tax		0	0	0	0	0	0	0	26	172
EBIAT		–130	–100	–50	–25	0	70	150	224	258
Less: Ch. NWC		10	4	7	4	4	9	12	15	25
Free Cash Flow		–140	–104	–57	–29	–4	61	138	209	233
Discount Factor		0.870	0.756	0.658	0.572	0.497	0.432	0.376	0.327	0.284
PV (Cash Flow)		–122	–79	–37	–17	–2	26	52	68	66
PV (Cash Flows)	(44)									
Terminal Value										2000
PV (Terminal Value)										569

Net Present Value and Sensitivity Analysis

PV (Cash Flows)	(44)
PV (Terminal Value)	569
Net Present Value	525

		WACC		
		13%	15%	17%
Terminal	2%	699	476	323
Growth	3%	778	525	355
Rate	4%	876	583	391

Tax Calculation	1	2	3	4	5	6	7	8	9
EBIT	–130	–100	–50	–25	0	70	150	250	430
NOLs Used	0	0	0	0	0	70	150	185	0
NOLs Added	130	100	50	25	0	0	0	0	0
Tax	0	0	0	0	0	0	0	26	172
Beginning NOLs	100	230	330	380	405	405	335	185	0
Ending NOLs	230	330	380	405	405	335	185	0	0

Net Working Capital (10% sales)	1	2	3	4	5	6	7	8	9
Beg NWC		10	14	21	25	29	38	50	65
End NWC	10	14	21	25	29	38	50	65	90
Ch. NWC	10	4	7	4	4	9	12	15	25

Appendix 3
Sample Valuation Using the Adjusted Present Value Method

Vulture Partners, a private equity organization specializing in distressed company investing, was interested in purchasing Turnaround. Mr. Fang, a general partner at Vulture, used the following projections to value Turnaround (all data are in millions of dollars):

	Year 1	Year 2	Year 3	Year 4	Year 5
Revenues	200	210	220	230	240
Costs	100	105	110	115	120
EBIT	100	105	110	115	120
∆NWC	3	3	4	4	5

Turnaround had $220 million of NOLs which were available to be offset against future income. At the beginning of Year 1, the company had $75 million of 8% debt which was expected to be repaid in three $25 million installments, beginning at the end of Year 1. The tax rate was 40%. Mr. Fang believed an appropriate unlevered beta for Turnaround was 0.8. The ten-year Treasury Bond yield was 7.0% and the market risk premium 7.5%. Net cash flows were forecast to grow at 3% per year in perpetuity after Year 5. Mr. Fang performed the following steps.

Mr. Fang employed the APV method to value Turnaround and, as such, used the cost of equity as the discount rate:

$$\text{Cost of Equity} = r_f + b_u * (r_m - r_f) = 7.0 + 0.8 * (7.5) = 13.0\%$$

Cash flows and the terminal value were both calculated in the same manner as under the NPV method. Mr. Fang arrived at a terminal value of $690 million using the perpetuity method (assuming a growth rate of 3% per annum).

Mr. Fang then calculated the interest tax shields by multiplying the interest expense for each period by the tax rate of 40%. The interest expense was calculated using the debt repayment schedule. The present value of the interest tax shields, equal to $4.2 million, was determined by discounting each year's interest tax shield at the pre-tax cost of debt.

To value the tax shields from the NOLs, Mr. Fang first determined the taxable earnings for each period and hence the rate at which the NOLs would be utilized. By subtracting the interest expense on debt from taxable earnings (EBIT), he determined the amount of NOLs that would be used each period. The NOL tax shields were then calculated by multiplying the NOLs consumed each period by the tax rate. Mr. Fang discounted the NOL tax shields at the pre-tax cost of debt. The present value of the NOLs was equal to $77 million.

The sensitivity analysis showed the likely valuation range for Turnaround to be on the order of $650 to $750 million. The range of values indicated the valuation was reasonably sensitive to both the discount and terminal growth rate assumptions.

Appendix 3 (continued)

Discount Rate Calculation			
Tax Rate	40%	Rm – Rf	7.5%
10 Year Treasury Bond	7.0%	Bu	0.8
Discount Rate (Unlevered)	13.00%		

Step 1: Value Cash Flows						
Terminal Growth Rate	3.00%					
Year	0	1	2	3	4	5
Revenues		200	210	220	230	240
Less: Costs		100	105	110	115	120
EBIT		100	105	110	115	120
Less: Tax		40	42	44	46	48
EBIAT		60	63	66	69	72
Less: Ch. NWC		3	3	4	4	5
Net Cash Flow		57	60	62	65	67
Discount Factor		0.885	0.783	0.693	0.613	0.543
PV (Cash Flow)		50	47	43	40	36
PV (Cash Flows)	217					
Terminal Value						690
PV (Terminal Value)						375

Step 2: Value Interest Tax Shields						
Beginning Debt		75	50	25	0	0
Repayment (End of Year)		25	25	25	0	0
Ending Debt		50	25	0	0	0
Interest Expense		6.0	4.0	2.0	0.0	0.0
Interest Tax Shield		2.4	1.6	0.8	0.0	0.0
Discount Factor	8.00%	0.926	0.857	0.794	0.735	0.681
Present Value		2.2	1.4	0.6	0.0	0.0
Net Present Value	4.2					

Step 3: Value NOLs						
EBIT		100	105	110	115	120
Interest Expense		6.0	4.0	2.0	0.0	0.0
EBIT less Interest Expense		94	101	108	115	120
NOLs Used		94	101	25	0	0
Beginning NOLs		220	126	25	0	0
Ending NOLs		126	25	0	0	0
NOLs Used		94	101	25	0	0
NOL Tax Shield		38	40	10	0	0
Discount Factor	8.00%	0.926	0.857	0.794	0.735	0.681
Present Value (NOL)		35	35	8	0	0
Net Present Value (NOLs)	77					

Step 4: NPV and Sensitivity Analysis	
PV (Cash Flows)	217
PV (Terminal Value)	375
PV (Tax Shields)	4
PV (NOLs)	77
Net Present Value	673

		WACC		
		12.0%	13.0%	14.0%
Terminal Growth Rate	2%	692	635	589
	3%	739	673	619
	4%	798	718	655

Appendix 4
Sample Valuation Using the Venture Capital Method

James is a partner in a very successful Boston-based venture capital firm. He plans to invest $5 million in a start-up biotechnology venture and must decide what share of the company he should demand for his investment. Projections he developed with company management show net income in year seven of $20 million. The few profitable biotechnology companies are trading at an average price-earnings ratio of 15. The company currently has 500,000 shares outstanding. James believes that a target rate of return of 50% is required for a venture of this risk. He performs the following calculations:

*Discounted Terminal Value = Terminal Value / (1 + Target)^{years} = (20 * 15) / (1+ 50%)^{7} = $17.5 million*

Required Percent Ownership = Investment / Discounted Terminal Value = 5 / 17.5 = 28.5%

Number of New Shares = 500,000 / (1 - 28.5%) - 500,000 = 200,000

Price per New Share = $5 million / 200,000 shares = $25 per share

*Implied Pre-money Valuation = 500,000 shares * $25 per share = $12.5 million*

*Implied Post-money Valuation = 700,000 shares * $25 per share = $17.5 million*

James and his partners are of the opinion that three more senior staff will need to be hired. In James's experience this number of top caliber recruits would require options amounting to 10% of the common stock outstanding. Additionally, he believes that, at the time the firm goes public, additional shares equivalent to 30% of the common stock will be sold to the public. He amends his calculations as follows:

Retention Ratio = [1 / (1 + .1)] / (1 + .3) = 70%

Required Current Percent Ownership = Required Final Percent Ownership / Retention Ratio = 28.5% / 70% = 40.7%

Number of New Shares = 500,000 / (1 - 40.7%) - 500,000 = 343,373

Price per New Share = $5 million / 343,373 shares = $14.56 per share

Appendix 5
Sample Valuation Using Option Pricing

Sharon Rock, a famous venture capitalist, was considering whether to invest in ThinkTank, Inc., a company owned and managed by Mr. Brain. ThinkTank had developed a new product that was ready to be manufactured and marketed. An expenditure of $120 million was required for the construction of research and manufacturing facilities. Rock was of the opinion that the following projections developed by Mr. Brain and his associates were justifiable (all data are in millions of dollars):

	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Cash Flow except CapEx	0.0	0.0	0.0	10.0	25.0	50.0
Capital Expenditures	–120.0	0.0	0.0	0.0	0.0	0.0
Total Cash Flow	–120.0	0.0	0.0	10.0	25.0	50.0

Rock performed a NPV valuation using a discount rate (WACC) of 25% and a terminal growth rate of 3%. She was unimpressed with the resulting valuation of –$11.55 million.

After thinking more carefully, Rock realized that the investment could be broken into two stages. The initial investment, which would need to be made immediately, would be $20 million for R&D equipment and personnel. The $100 million expenditure on the plant could be undertaken any time in the first two years. (Whenever the project would be undertaken, the present value of the plant construction expenditures would total $100 million in today's dollars.) Rock decided that the option to expand should not be valued using discounted cash flow methods, as she would only pursue the opportunity if the first stage of the project were successful. The expansion opportunity could more validly be considered as an initial $20 investment bundled with a two-year European call option and priced using the Black-Scholes model.

The easiest variables to estimate were the time to expiration (t) and the risk free rate (r_f), being 2 years and 7%, respectively. The "exercise price" (X) was equal to the present value of the investment to build the plant, or $100 million. The "stock price" (S) was estimated by discounting the expected cash flows to be generated by the underlying assets associated with the expansion opportunity. Using a discount rate of 25% and a terminal growth rate of 3% per year, S was calculated as worth as $108.45 million in Year 0. The only Black-Scholes input variable remaining to be calculated was the standard deviation (σ,). Rock found this difficult to estimate but proceeded to look at some comparable companies. She estimated that the value of σ, was likely to lie in the range of 0.5 to 0.6.

Using this data Rock then calculated the Black-Scholes European call option to be worth between $38.8 and $43.7 million. The total net present value of the project, equal to the cost of the first stage investment and the value of the call option (the stage 2 opportunity), was therefore between $18.8 and $23.7 million [= –$20 million + $38.8 to $43.7 million].

Based on this analysis, Sharon Rock decided to invest in ThinkTank on the provision that she would be granted first right of refusal on any subsequent rounds of financing.